Chapter 12 Static Equilibrium and Elasticity

Problems: 1, 3, 6, 19, 57 Bonus: 56, 60

1 True or false:

(a) $\sum_i \vec{F}_i = 0$ is sufficient for static equilibrium to exist.

(b) $\sum_i \vec{F}_i = 0$ is necessary for static equilibrium to exist.

(c) In static equilibrium, the net torque about any point is zero.

(d) An object in equilibrium cannot be moving.

(a) False. The conditions $\sum \vec{F} = 0$ <u>and</u> $\sum \vec{\tau} = 0$ must both be satisfied.

(b) True. The necessary and sufficient conditions for static equilibrium are $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$.

(c) True. The conditions $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$ must be satisfied.

(d) False. An object can be moving with constant speed (translational or rotational) when the conditions $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$ are satisfied. In static equilibrium the speed does not change.

3 The horizontal bar in Figure 12-27 will remain horizontal if (a) $L_1 = L_2$ and $R_1 = R_2$, (b) $M_1R_1 = M_2R_2$, (c) $M_2R_1 = R_2M_1$, (d) $L_1M_1 = L_2M_2$, (e) $R_1L_1 = R_2L_2$.

Determine the Concept The condition that the bar is in rotational equilibrium is that the net torque acting on it equal zero; $\sum \tau = R_1M_1 - R_2M_2 = 0$ $\boxed{(b)}$ is correct.

6 A father (mass M) and his son, (mass m) begin walking out towards opposite ends of a balanced see-saw. As they walk, the see-saw stays exactly horizontal. What can be said about the relationship between the father's speed V and the son's speed v?

Determine the Concept The question is about a situation in which an object is in static equilibrium. Both the father and son are walking outward from the center of the see-saw, which always remains in equilibrium. In order for this to happen, at any time, the net torque about any point (let's say, the pivot point at the center of the see-saw) must be zero. We can denote the father's position as X, and the son's position as x, and choose the origin of coordinates to be at the pivot point. At each moment, the see-saw exerts normal forces on the son and his father equal to their respective weights, mg and Mg. By Newton's third law, the father exerts a downward force equal in magnitude to the normal force, and the son exerts a downward force equal in magnitude to the normal force acting on him.

Apply $\sum \tau_{\text{pivot point}} = 0$ to the see-saw $\qquad MgX - mgx = 0 \qquad (1)$

(assume that the father walks to the left and that counterclockwise torques are positive):

Express the distance both the father and his son walk as a function of time:

$X = V\Delta t$ and $x = v\Delta t$

Substitute for X and x in equation (1) to obtain:

$$MgV\Delta t - mgv\Delta t = 0 \Rightarrow V = \boxed{\frac{m}{M}v}$$

Remarks: The father's speed is less than the son's speed by a factor of *m*/*M*.

16 Figure 12-30 shows a lever of negligible mass with a vertical force F_{app} being applied to lift a load F. The *mechanical advantage* of the lever is defined as $M = F/F_{app,\,min}$, where $F_{app,\,min}$ is the smallest force necessary to lift the load F. Show that for this simple lever system, $M = x/X$, where x is the moment arm (distance to the pivot) for the applied force and X is the moment arm for the load.

Picture the Problem We can use the given definition of the mechanical advantage of a lever and the condition for rotational equilibrium to show that $M = x/X$.

Express the definition of mechanical advantage for a lever:

$$M = \frac{F}{F_{app,\,min}} \qquad (1)$$

Apply the condition for rotational equilibrium to the lever:

$$\sum \tau_{fulcrum} = xF_{app,\,min} - XF = 0$$

Solve for the ratio of F to $F_{app,\,min}$ to obtain:

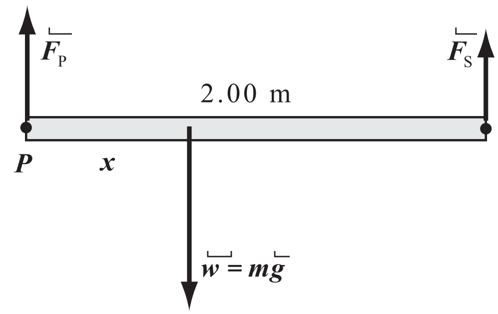
$$\frac{F}{F_{app,\,min}} = \frac{x}{X}$$

Substitute for $F/F_{app,\,min}$ in equation (1) to obtain:

$$M = \boxed{\frac{x}{X}}$$

19 A *gravity board* is a convenient and quick way to determine the location of the center of gravity of a person. It consists of a horizontal board supported by a fulcrum at one end and a scale at the other end. To demonstrate this in class, your physics professor calls on you to lie horizontally on the board with the top of your head directly above the fulcrum point as shown in Figure 12-33. The scale is 2.00 m from the fulcrum. In preparation for this experiment, you had accurately weighed yourself and determined your mass to be 70.0 kg. When you are at rest on the gravity board, the scale advances 250 N beyond its reading when the board is there by itself. Use this data to determine the location of your center of gravity relative to your feet.

Picture the Problem The diagram shows $\vec{w}$, the weight of the student, $\vec{F}_P$, the force exerted by the board at the pivot, and $\vec{F}_s$, the force exerted by the scale, acting on the student. Because the student is in equilibrium, we can apply the condition for rotational equilibrium to the student to find the location of his center of gravity. We ignore the mass of the board since $\vec{F}_s$ was taken after the scale was "zeroed" with the board in place.



Apply $\sum \vec{\tau} = 0$ about an axis through the pivot point P:

$$F_s(2.00\,\text{m}) - wx = 0$$

Solving for x yields:

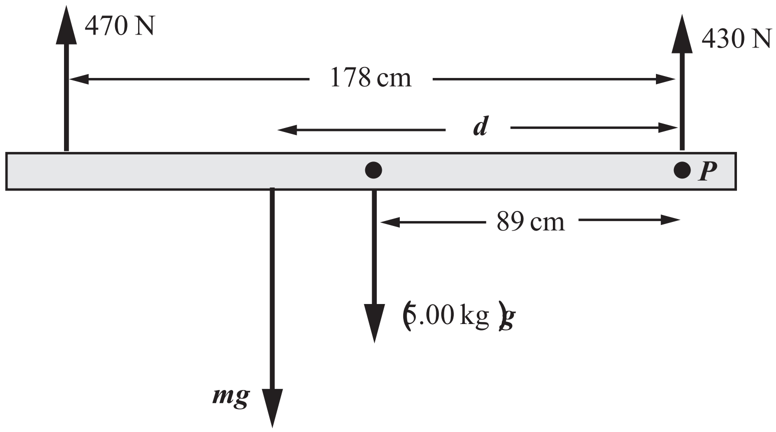
$$x = \frac{(2.00\,\text{m})F_s}{w}$$

Substitute numerical values and evaluate x:

$$x = \frac{(2.00\,\text{m})(250\,\text{N})}{(70.0\,\text{kg})(9.81\,\text{m/s}^2)} = \boxed{0.728\,\text{m}}$$

56 •• A biology laboratory at your university is studying the location of a person's center of gravity as a function of their body weight. They pay well, and you decide to volunteer. The location of your center of gravity when standing erect is to be determined by having you lie on a uniform board (mass of 5.00 kg, length 2.00 m) supported by two scales as shown in Figure 12-54. If your height is 188 cm and the left scale reads 470 N while the right scale reads 430 N, where is your center of gravity relative to your feet? Assume the scales are both exactly the same distance from the two ends of the board, are separated by 178 cm, and are set to each read zero before you get on the platform.

Picture the Problem Because the you-board system is in equilibrium, we can apply the conditions for translational and rotational equilibrium to relate the forces exerted by the scales to the distance d, measured from your feet, to your center of mass and the distance to the center of gravity of the board. The following pictorial representation shows the forces acting on the board. mg represents your weight.



The forces responsible for a counterclockwise torque about an axis through your feet (point P) and perpendicular to the page are your weight and the weight of the board. The only force causing a clockwise torque about this axis is the 470 N force exerted by the scale under your head. Apply $\sum\vec{\tau} = 0$ about an axis through your feet and perpendicular to the page:

$$\boldsymbol{m}\left(9.81\ \text{m/s}^2\right)\boldsymbol{d} - (5.0\ \text{kg})(0.89\ \text{m}) - (1.78\ \text{m})(470\ \text{N}) = 0$$

where m is your mass.

Solve for d to obtain:

$$\boldsymbol{d} = \frac{(5.00\ \text{kg})(0.89\ \text{m}) + (1.78\ \text{m})(470\ \text{N})}{\boldsymbol{m}\left(9.81\ \text{m/s}^2\right)} \qquad (1)$$

Let upward be the positive y direction and apply $\sum F_y = 0$ to the plank to obtain:

$$470\ \text{N} + 430\ \text{N} - \boldsymbol{m}\left(9.81\ \text{m/s}^2\right) - (5.00\ \text{kg})\left(9.81\ \text{m/s}^2\right) = 0$$

Solving for m yields: $\boldsymbol{m} = 86.74\ \text{kg}$

Substitute numerical values in equation (1) and evaluate d:

$$\boldsymbol{d} = \frac{(5.0\ \text{kg})(0.89\ \text{m}) + (1.78\ \text{m})(470\ \text{N})}{(86.74\ \text{kg})\left(9.81\ \text{m/s}^2\right)} = \boxed{99\ \text{cm}}$$

57 •• Figure 12-49 shows a mobile consisting of four objects hanging on three rods of negligible mass. Find the values of the unknown masses of the objects if the mobile is to balance. *Hint: Find the mass m_1 first.*

Picture the Problem We can apply the balance condition $\sum\vec{\tau} = 0$ successively, starting with the lowest part of the mobile, to find the value of each of the unknown weights.

Apply $\sum \vec{\tau} = 0$ about an axis through the point of suspension of the lowest part of the mobile:

$$(3.0\,\text{cm})(2.0\,\text{N}) - (4.0\,\text{cm})m_1 g = 0$$

Solving for m_1 yields:

$$m_1 = 0.1529\,\text{kg} = \boxed{0.15\,\text{kg}}$$

Apply $\sum \vec{\tau} = 0$ about an axis through the point of suspension of the middle part of the mobile:

$$(2.0\,\text{cm})m_2 g - (4.0\,\text{cm})\left(\frac{2.0\,\text{N}}{g} + 0.1529\,\text{kg}\right)g = 0$$

Solving for m_2 yields:

$$m_2 = 0.7136\,\text{kg} = \boxed{0.71\,\text{kg}}$$

Apply $\sum \vec{\tau} = 0$ about an axis through the point of suspension of the top part of the mobile:

$$(2.0\,\text{cm})(2.0\,\text{N} + (0.7136\,\text{kg})g + (0.1529)g) - (6.0\,\text{cm})m_3 g = 0$$

Solving for m_3 yields:

$$m_3 = 0.3568\,\text{kg} = \boxed{0.36\,\text{kg}}$$

60 •• A rope and pulley system, called a *block and tackle*, is used to raise an object of mass *M* (Figure 12-52) at constant speed. When the end of the rope moves downward through a distance *L,* the height of the lower pulley is increased by *h*. (*a*) What is the ratio *L*/*h*? (*b*) Assume that the mass of the block and tackle is negligible and that the pulley bearings are frictionless. Show that *FL* = *mgh* by applying the work–energy principle to the block–tackle object.

Picture the Problem We can determine the ratio of *L* to *h* by noting the number of ropes supporting the load whose mass is *M*.

(*a*) Noting that three ropes support the pulley to which the object whose mass is *M* is fastened we can conclude that:

$$\frac{L}{h} = \boxed{3}$$

(*b*) Apply the work-energy principle to the block-tackle object to obtain:

$$W_{\text{ext}} = \Delta E_{\text{system}} = \Delta U_{\text{block-tackle}}$$

or

$$FL = \boxed{mgh}$$